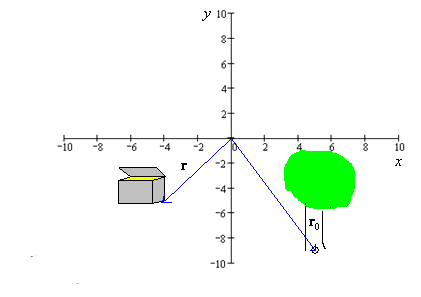
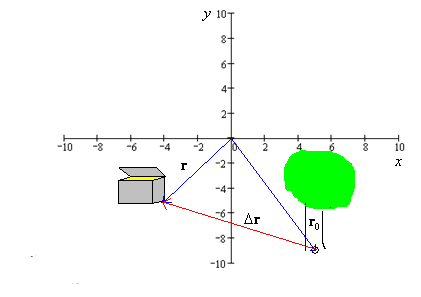
**Example: Finding Δr given r0 and r**

Suppose we’re trying to find buried treasure. We start at a palm tree located at the coordinate (5 miles, -9 miles). And the map tells us that the treasure is located at the coordinate (-4 miles, -5 miles). Then how far and in what direction must we travel in order to get to the treasure from the tree?

To answer the question we first plot our initial and final positions. Our initial position is . And our final position is , illustrated below.



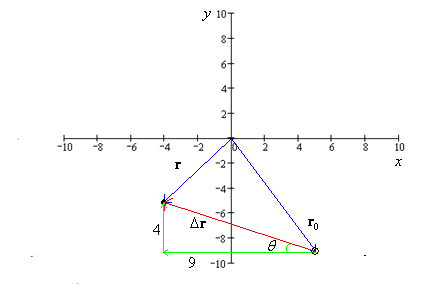
Then our displacement vector, which we’re trying to find, goes from our initial to final position,



and we should recognize that,



So we should go 9 miles west, and 4 miles northward, as illustrated below,



Alternatively we may ask for the magnitude and direction of our displacement vector, Δ**r**. The magnitude and direction of Δ**r** is

,

 north of west.

And so also,



meaning we would travel 9.85 miles at an angle 24˚ north of west, starting from the tree to get to the treasure.

1. Alexander the Great sets out from city A, marching his Macedonian army a distance of 150 miles, 20 degrees north of east, to city B. Then he marches to city C to meet the Persians in battle, traversing 80 miles, at an angle of 50 degrees south of west. What is the magnitude and direction of his total displacement from A to C.

First displacement can be written as:



Second displacement is:



So net displacement is:

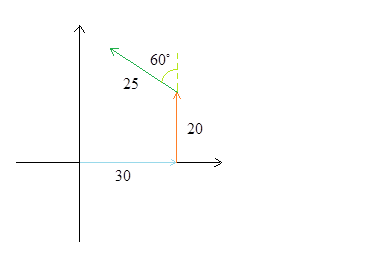


separating into magnitude and direction we have:

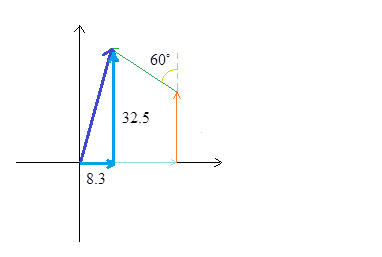


**Question 1. You obtain a secret copy of Alexander the Great’s battle plan. Suppose he plans to march his army East for 30 miles, then North for 20 miles, and then finally 25 miles in direction 60° W of N. Draw a picture of his path, and then mathematically determine what single displacement vector you could make to meet him there, starting from the same point he did. Give this vector’s magnitude and direction.**

**A picture of the movements is given below:**



**The first displacement can be written d1 = (30,0). The second as d2 = (0,20). And the third as d3 = (-25sin60, 25cos60) = (-21.7, 12.5). Adding these together gives the net displacement vector d = d1 + d2 + d3 = (30,0) + (0,20) + (-21.7,12.5) = (8.3,32.5). Drawing the net displacement vector below, on the same graph just to evince its relationship to the other three vectors, we have:**



**The magnitude and direction of this vector is:**



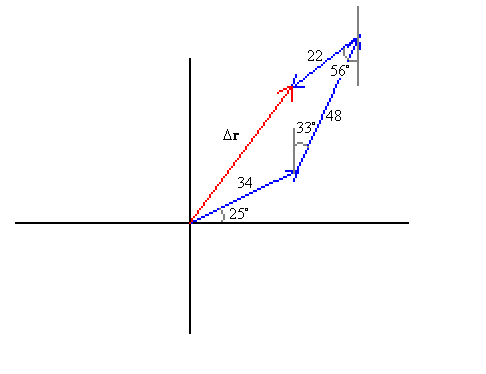
1. Suppose Tim and Percy start at the same spot. Tim goes to the left 2m and north 3m. Percy on the other hand runs to the right 10m, and then south 3m. How far and in what direction should Tim throw the football so that Percy will catch it?

The displacement vector of the ball is:



**P2.** Car travels 34 miles 25 degrees N of E, then 48 miles 33 degrees East of North, and then 22 miles 56 degrees West of South. What is their net displacement magnitude and direction?

The displacements are shown below, along with the net displacement vector in red.



Going counter clockwise, the net displacement is:



which has a magnitude of in a direction

 North of East.

2. A high speed camera takes pictures of the position of a motorcycle at a rate of 2400 pictures per minute. If the motorcycle has a speed of 100 km/hr, how far does it travel in between frames?

The distance traveled follows from the formula x = x­0 + v0xt + (1/2)axt2. We take x0 = 0, and ax is zero as well since the motorcyle is traveling at a constant rate. Therefore we have: x = v0xt. Now v0x = 100 km/h, and the t is the time between frames, t = min/2400. Multiplying these together we have:



5. A plane flies 200 m/s north for 5 minutes, and then 300m/s east for 2 minutes. What is the magnitude of the plane’s average velocity?

One way to do this is the following…The plane’s displacement is:



The total time is:



So the average velocity vector is:



and its magnitude is:



**P8.** It takes 3 hours 20 minutes to travel to your home from vacation. The first segment of the trip is 130km, during which you drive 95 kph, and the second you drive at 65 kph. How far away was your home? And what was your average velocity?

The distance to home is the sum of your two displacements. The first we know is 130km (to the right, say). And the second is unknown, but we know the velocity so we’ll write it in terms of the velocity.



So now we need Δt2. We can get it by the following consideration. We know that the total time is 3 hours 20 minutes = 3 hours + 20/60 hours = 3.33 hours. The time for the first part of the trip is Δt1 = Δr1/v1 = 130km/95km/h = 1.37h. So we can solve for the time for the second part of the trip.



So filling this into our equation for Δr, we get,



so we drove 257 km. The average speed is distance/time which is:



9. One runner is at position x0 = -10m. Runner two is at position x0 = 5m. If runner one accelerates towards runner two at rate a1 = 1m/s2 and runner two accelerates towards runner 1 at rate a2 = -3m/s2, where do they meet?

We write the position of runner 1 and 2 as a function of time,



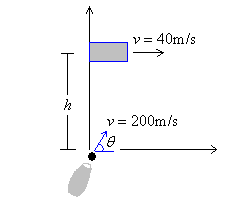
They will meet when,



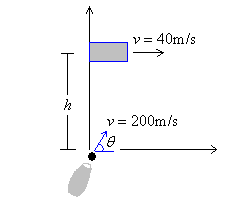
The position will be:



10. A really fast tank is traveling to the right at speed 40m/s, at initial position h = 500m on the y-axis. Just as the tank crosses the y-axis, you fire a projectile (muzzle velocity v = 200m/s) at angle θ to hit the tank. At what angle θ should you aim the gun?



We have



We use the x and y equations for the position of the projectile. Note that we assume ax = ay = 0. So then…



the position of the tank is:



In order for these to meet we need the positions to be the same. So this requires xp = xt and yp = yt. Turns out we only need the x equation…

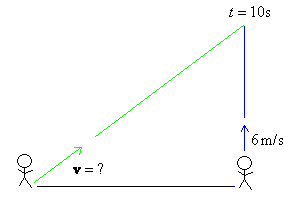


The y-equation would tell us when they meet, but we don’t care for this problem.

**Question 6**

Suppose your friend Fred is 20m east from you. He starts running north with a velocity of 6m/s, and you immediately pursue him. If you want to catch him in 10s, what should your velocity vector be? Give its magnitude and direction. You may assume that everything is at constant velocity.

The situation is diagrammed below,



The position of you as a function of time is:



where we set the origin of our coordinate system to be at your position. The position of Fred as a function of time is:



You want to meet him in 10s therefore we must have **r**you(10) = **r**Fred(10). This means,



The magnitude and direction of **v** are:



7. Fred and Bob are competing in a marathon. Suppose that near the end of the race, Bob is 30m before the finish line and running with a speed of 6m/s, while Fred is 20m before the finish line running with a constant speed of 7m/s. How quickly must Bob accelerate to catch Fred by the end of the race?

The position of Fred as a function of time is:



And Fred will be arrive at the finish line in a time t given by:



The position of Bob as a function of time is:



In order for him to catch Fred at the finish line, he must get to x = 0 in 2.86s, which means,



9. Suppose you’re trying out for the football team. You run 50m in 4.6s, starting from rest, accelerating at a constant rate. What was your final velocity?

Use the equation,



Let your initial position be 0. Then your final position is 50. Your initial velocity is 0, and your final is unknown. Plugging in all this information we have, leaving off the vector notation since its just 1 dimension,



10. In the problem before, what was your constant acceleration?

Your acceleration would be,



**P11.** Suppose your initial position is , and initial velocity is , and acceleration is  What is your position 1s later? What is your position 2s later? When do you turn around? How far will you have traveled by then? When do you arrive back at your starting point?

Your position as a function of time will be:



So after 1s your position will be



And after 2s your position will be



You will turn around when your velocity is 0. Your velocity as a function of time is:



Your velocity will be 0 when:



so after 2.5s.

At this point your position would be:



and so you would have travelled a distance,



You would arrive back at your starting point when , the initial position So we equate,



So you’ll return to your original position after 5s.

**Problem 3**

You are driving along the highway at 25m/s, when you see a deer 15m ahead of you. What constant acceleration would be required to stop in those 15m?

We can figure out the time from the velocity equation:



and then plug this time into the position equation:



**Question 5**. Driving down the highway at 25m/s you suddenly spot a dear 40m away. If your reaction time is 0.20s, what deceleration will bring you to rest by the time you reach the dear’s location?

Before you start decelerating, you will have traveled a distance x = x0 + v0xt + (1/2)axt2 = 0 + 25(0.20) + (1/2)(0)(0.20)2 = 5m. So then you’ll need to accelerate so that, using the same equation as above, 40 = 5 + (25)t + (1/2)at2 🡪 35 = 25t + (1/2)axt2. And from the velocity equation we also have vx = v0x + axt 🡪 0 = 25 + axt 🡪 ax = -25/t. Plugging this into the position equation, we have: 35 = 25t - (1/2)(25/t)t2 🡪 35 = 25t - 12.5t 🡪 35 = 12.5t 🡪 t = 35/12.5 = 2.8s. And so ax = -25/2.8 = -8.9 m/s2

**Question 5.** You’re driving your car at a speed of v = 30m/s, when you notice that 200m ahead there is a cliff. With what acceleration must you break in order to come to rest by the time you get to the edge of the cliff?

The position equation is given by:



Letting x0 = 0, x = 200, v0x =30 we have:



The velocity equation is



Filling in vx = 0, v0x = 30 we have:



Solve for t and plug into the x equation to get:



4a. A car is initially at position x0 = 10m, and has initial velocity v0 = 30m/s. Suppose the driver slams on the brakes, decelerating the car at a constant rate of 5m/s2. Write down an expression for the velocity and position of the car as a function of time.



4b. What will be the car’s velocity and position 3s later?



4c. How long (time-wise) will it take for the car to stop? How far will it have traveled by then?

It will stop when…



and the distance traveled at that point will be:

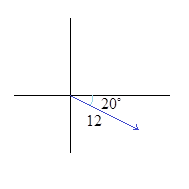


4d. When will the car have reached the position x = 100m?

It will have reached 100m, when t = 6s. That was easier than I anticipated.

5a. A drone is at vertical position y0 = 1000m above the ground traveling with an initial velocity **v**0 = 120m/s East. Then the drone operator gives it a constant acceleration of **a** **=** 12m/s2 @ 20° below the horizontal. Write down an expression for vx, vy, x, and y as a function of time (you can treat the initial x position as being x0 = 0), letting x be the Eastward direction and y be the vertical (upwards) position.

The initial velocity vector is given by **v**0 = (120,0). Therefore v0x = 120m/s and v0y = 0m/s. The acceleration vector looks like this:



and so it can be written as **a** = (12cos20°,-12sin20°) = (11.3,-4.1). This identifies ax as 11.3m/s2, and ay as -4.1 m/s2. And so now we can write:



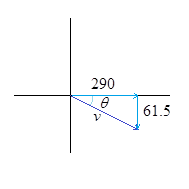
5b. What will be its position (x,y) and velocity (vx,vy) fifteen seconds later?

So its position and velocity will be:



5c. What will be the magnitude and direction of its velocity at that time?

The velocity vector will look like this:

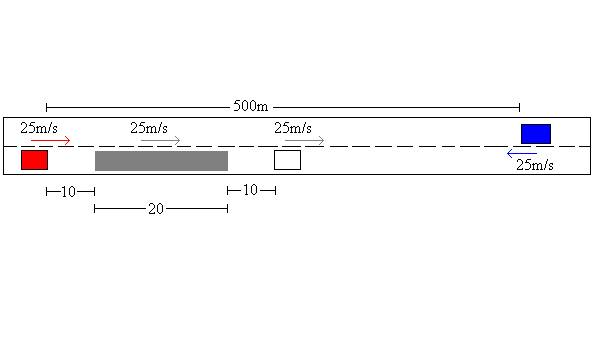


and so its magnitude and direction will be:



**P14.** Problem 2.30 in the text.

The situation is illustrated below. The empty square in front of the truck is the spot the red car wants to get to before meeting the blue car.



So the red car has an initial position of say, 0, and is traveling at an initial velocity of 25m/s, and an acceleration of 1m/s2. So its position as a function of time is:



The spot it is trying to get to has an initial position of 40m, and is traveling with a velocity of 25m/s (since the spot remains ahead of the truck which is traveling at that velocity as well) with no acceleration. So its position as a function of time is:



The blue car has an initial position of 500m and is traveling at a velocity of -25m/s with no acceleration. So its position as a function of time is:



The question is whether the red car can catch up to the spot before the blue car reaches it. So what we do is determine when the red car gets to the spot, from that time determine where the red car catches up to the spot, and then see also where the blue car is at that time. So first, the red car will catch up to the spot when their positions are equal.



Where will the red car be at that point? We plug into its position formula to determine this:



Where is the blue car?



So the red car is at the position x = 264, and the blue car at the position x = 276. Therefore the red car will still be to the left of the blue car (so they will not have crossed yet), and the red car will have just barely made it to the spot before the blue car gets there.

**Question 3**

Suppose you’re trying out for the football team. You run 50m in 4.6s, starting from rest, by accelerating up to a constant velocity by the 25m point, and then running at that constant velocity for the next 25m. What was your initial acceleration? And what was the constant velocity?

For the first half of the motion we have the distance traveled is 25m, we’ll take the time to be t1, and the acceleration to be a.



For the second half of the motion, the distance traveled is 25m, the initial velocity is v = at1, and the time to traverse the distance we’ll call t2. So we have,



Let’s equate these two expressions and we’ll get,



Now finally use the fact that, t1 + t2 = 4.6s. So, t1 = 4.6 – t2. Substituting this in we get,



And so,



From equation (1) we get,



and the constant velocity over the second half is:



**Question 4.**

Suppose the acceleration of a car along the x-axis is given by . If the car starts with an initial speed v0x of 10m/s, at the initial position x0 = 0, what will be its position 5s later?

Integrate twice to get the position. We have,



Then integrate once more,



So the position is given by:



and at t = 5, the position will be:



**Problem 1.**

A bird is taking flight. It’s position vector is given by **r**(t) = 12t**i** + 30(1 – e −t/2)**j**, where **i** and **j** point East and up respectively. (a) What is the magnitude and direction of the bird’s velocity at time t = 2s?



(b) What is the magnitude and direction of the bird’s acceleration at t = 2s?

Taking another derivative,



**Problem 2**

At time t = 0, a car’s position is x = 100m, its velocity is vx = 27m/s, and its acceleration is ax = −2t. (a) What is the car’s position, x(t), has a function of time?

The velocity is:



and the position is:



(b) When does the car stop?

The car stops when vx = 0, i.e., when



2. A particle’s acceleration is given by: . If the initial velocity is , and initial position is: , what are the x and y coordinates of the particle at time t = 5s?

Integrating once to get velocity…



Now integrating again to get position…



Filling in the time…



3. A particle’s position is described by the vector: . What is the magnitude and direction of the particle’s velocity at time t = 5s? Give the angle w/r to the positive x axis.

The velocity is given by:



and so at t = 5 we have:



5. You’re driving your car at a speed of v = 30m/s, when you notice that 200m ahead there is a cliff. With what acceleration must you break in order to come to rest by the time you get to the edge of the cliff?

The position equation is given by:



Letting x0 = 0, x = 200, v0x =30 we have:



The velocity equation is



Filling in vx = 0, v0x = 30 we have:



Solve for t and plug into the x equation to get:



2. You’re running along the ground such that your position is given by  with t starting at 0. What is the magnitude and direction of your velocity when you cross the x-axis the first time? Don’t forget that cos(πt) is to be evaluated in radian mode.

You will cross the x-axis for the first time when,



Your velocity at any given time is:



and so your velocity at t = 1/π seconds is:



3. The acceleration of a car along the *x*-axis is given by ax = 5e-t. The car starts with an initial speed v0 of 6.5m/s, at the initial position x0 = -3m. What will be its position 5s later?

The velocity as a function of time is:



and the position as a function of time is:



So the position at t = 5s is:



**Question 8**. Suppose that you step on the accelerator and your car accelerates forward at a rate ax = 200/(5+t)2. (a) What will be your speed at t = 10s? (b) How far will you have gone?

(a) We have:



Plugging in the time we get:



and your distance will be given by:



Plugging in the time…



**Question 5.**

Suppose your position is given by . What is the magnitude and direction of your acceleration at time t = 1s? Don’t forget that sin(t) is to be evaluated in radian mode.

The acceleration is given by the second derivative of the position vector w/r to time,



And so,



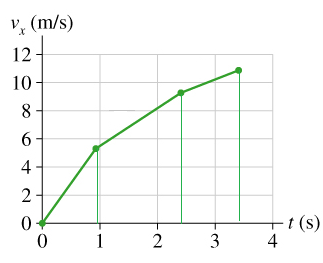
**Question 4.** A bird is taking flight. Starting at the position x = 0, y = 10, its velocity vector is given by **v**(t) = (1-e-t)**i** - t **j**, where **i** and **j** point east and up respectively. (a) What is the bird’s position vector as a function of time? (b) What is the bird’s acceleration vector as a function of time?



Taking another derivative,



**Question 4**. Carl Lewis’s velocity as a function of time during part of a 100m race is graphed below. (a) What was his initial acceleration? (b) Approximately how far had he run by the end of the graph (t ≈ 3.5s)

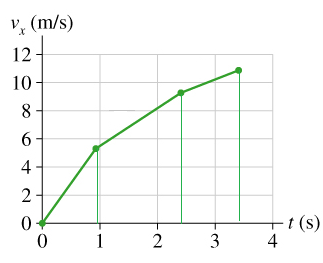


(a) Initial acceleration is given by slope the first segment. And this approximately ax = dvx/dt = (5.5m/s – 0m/s)/(1s – 0s) = 5.5m/s2.

(b) Distance travelled is area under curve from 0 to 3.5s. We can get the area by splitting it up into three trapezoids, as illustrated above. So we have approximately:



**Question 1**. Carl Lewis’s velocity as a function of time during part of a 100m race is graphed below. (a) What was his acceleration during the middle segment? (b) Approximately how far had he run by the end of the graph (t ≈ 3.5s)

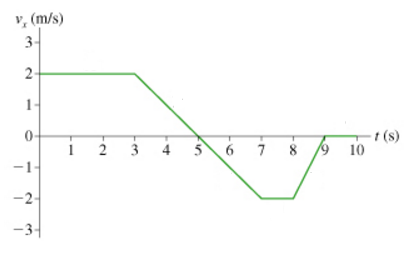


(a) His acceleration during the middle is given by the slope of the middle segment. And this approximately ax = dvx/dt = (9.5m/s – 5.5m/s)/(2.5s – 1s) = 2.67m/s2.

(b) Distance travelled is area under curve from 0 to 3.5s. We can get the area by splitting it up into three trapezoids, as illustrated above. So we have approximately:



**Question 4**. A slightly inebriated college student (over 21) stumbles around per the following velocity vs. time graph. (a) when does he turn around? (b) What is his largest acceleration during the 10s interval? (c) By 10s, how far away is he from his starting point?



Part a) turns around at t = 5s.

Part b) largest acceleration corresponds to the largest slope. This is at during the segment between t = 8s and 9s. The acceleration here is a = 2m/s2.

Part c) displacement is just area under the curve and this is: Δx = (2m/s)(3s) + (1/2)(2m/s + -2m/s)(4s) + (-2m/s)(1s) + (1/2)(-2m/s)(1s) = 3m. So he’s 3m to the right of his starting point.